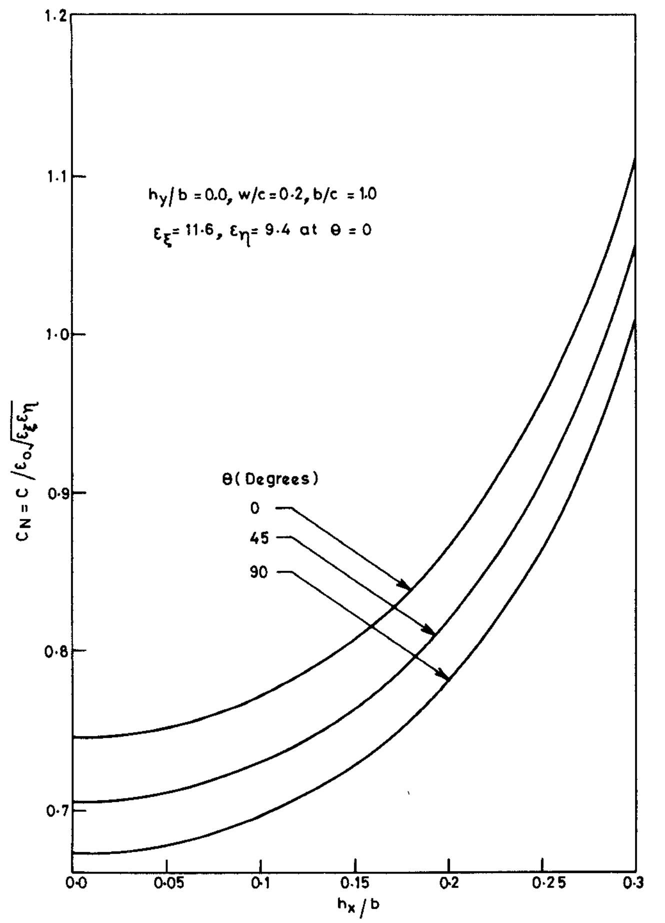

Fig. 5. Variation of the normalized capacitance C_N of square inhomogeneous coaxial line with sapphire dielectric as a function of edge-offset h_x/b with θ as the parameter.

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Incremental Frequency Rule for Computing the Q-Factor of a Shielded TE_{omp} Dielectric Resonator

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Abstract —**The principle of Wheeler's incremental inductance rule is applied to the TE_{omp} cylindrical resonator within a metal enclosure. The procedure permits one to compute the conductor losses solely from the decrease in resonant frequency when the metal walls are receded for one skin depth.**

I. INTRODUCTION

Computation of the Q-factor of metal cavities requires an integration of the dissipated power over the entire metal surface of the cavity. When a dielectric resonator is placed within the metal enclosure, analytical expressions for the field distribution become quite involved, and the numerical evaluation of Q-factor frequently requires various simplifying assumptions in order to render the solution possible [1], [2].

It is well known that the computation of conductor losses on the TEM transmission lines may be considerably simplified by using the "incremental inductance rule" developed by Wheeler [3]. This rule replaces the detailed surface integration by a simple computation of the increment in inductance per unit length when all the metal walls are receded by $\delta/2$, where the skin depth δ is given by

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}. \tag{1}$$

In the above, f is the frequency of operation, σ is the conductivity, and μ is the permeability of the metal walls.

It will be shown here that a similar trick can be applied also to the TE_{omp} modes in rotationally symmetric hollow resonators, but the increment which is to be calculated is now the increment in the resonant frequency.

II. THE RULE

The Q-factor due to conductor losses of any cavity consisting of a rotationally symmetric metal enclosure, supporting the TE_{omp}-type field, can be computed as follows:

$$Q_c = \frac{f_0}{\Delta f_0(\delta)}. \tag{2}$$

In the above, f_0 is the resonant frequency of the cavity, computed for the case when the metal enclosure is made of a perfect conductors. $\Delta f_0(\delta)$ is the increment in the resonant frequency, computed again for perfectly conducting walls which are now moved inwards for one full skin depth δ, evaluated by (1).

III. PROOF

Fig. 1 depicts a cylindrical dielectric resonator within a metal enclosure. When the enclosure is made of a perfect conductor, the knowledge of the magnetic-field intensity as function of position permits one to calculate the total stored magnetic energy W_m. When the enclosure is made of a conductor with finite conductiv-

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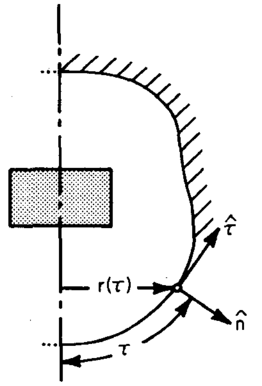

Fig. 1. Rotationally symmetric metal cavity containing a dielectric resonator.

ity σ, it is assumed that the tangential magnetic field on the metal surface $H_{\tau 0}(\tau)$ is the same as the one which existed previously on the perfect conductor. τ is the length along the circumference of the metal as shown in Fig. 1. Furthermore, an assumption is made that the local radius of curvature of the conducting surface is much larger than the skin depth at the frequency of operation. Then, the magnetic field distribution in the direction $\hat{n}$ normal to the metal surface is given by the skin-effect law [4]

$$H_\tau(\tau,n) = H_{\tau 0}(\tau) e^{-(1+j)n/\delta}. \tag{3}$$

Due to the nonvanishing field within conducting walls, the total magnetic energy is now increased by an amount W_{mi}, the magnetic energy within the conductor. For the mode with no azimuthal variation, the magnetic energy increment is:

$$W_{mi} = \int_{n=0}^{\infty} \int_{\tau=0}^{\tau_m} \frac{\mu}{4} |H_\tau(\tau,n)|^2 2\pi r(\tau)\, dn\, d\tau. \tag{4}$$

The integration in n can be readily evaluated by using (3)

$$\int_0^\infty H_\tau H_\tau^* dn = \frac{\delta}{2} |H_{\tau 0}(\tau)|^2. \tag{5}$$

The total integral of the exponentially decaying magnetic field is the same as if the field was constant within one-half of the skin depth δ, and vanishing beyond that distance. The increment of the stored magnetic energy then becomes

$$W_{mi} = \Delta W_m\left(\frac{\delta}{2}\right) = \frac{\mu}{4} \cdot \frac{\delta}{2} \int_{\tau=0}^{\tau_m} |H_{\tau 0}(\tau)|^2 2\pi r(\tau)\, d\tau. \tag{6}$$

The surface resistivity due to skin effect is given by [4]

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}}. \tag{7}$$

The power dissipated in the metal wall of finite conductivity is computed by integrating $\frac{1}{2} R_s |H_{\tau 0}|^2$ over the entire surface [4]

$$P_d = \frac{1}{2} R_s \int_{\tau=0}^{\tau_m} |H_{\tau 0}(\tau)|^2 2\pi r(\tau)\, d\tau \tag{8}$$

where τ_m is the total length of the arc describing the rotationally symmetric cavity in Fig. 1. It is seen that the integral appearing in (8) is the same as the one in (6). Therefore, the incremental magnetic energy is directly proportional to the dissipated power as follows:

$$P_d = 2\omega \Delta W_m\left(\frac{\delta}{2}\right) = \omega \Delta W_m(\delta). \tag{9}$$

$\Delta W_m(\delta/2)$ denotes the stored magnetic energy in the conductor, such as given by (6). According to (5), this energy may be obtained by integrating the squared surface magnetic-field intensity within a layer of thickness $\delta/2$. Since δ is very small in comparison with cavity dimensions, an integration over twice as large a thickness will give $\Delta W_m(\delta)$, which is twice the value of $\Delta W_m(\delta/2)$.

The Q-factor due to conductor losses is defined as

$$Q_c = \frac{\omega W_m}{P_d}. \tag{10}$$

In view of (9)

$$Q_c = \frac{W_m}{\Delta W_m(\delta)} \tag{11}$$

where W_m is the total energy stored in the cavity, and $\Delta W_m(\delta)$ is the incremental magnetic energy obtained by receding the cavity walls for one skin depth δ.

This brings us to the perturbation of cavity walls [5], [6]. A movement of walls in the opposite direction, namely into the cavity, would cause the resonant frequency f_0 to increase by an amount Δf_0, given by [7]

$$\frac{\Delta f_0(\delta)}{f_0} = \frac{\Delta W_m - \Delta W_e}{W_m}. \tag{12}$$

ΔW_e is the change in electric energy when the walls are moved in for δ. For the TE_{omp} mode, the normal component of the electric field on the cavity wall is zero, in addition to the tangential component of the electric field being zero, anyway, on the metal surface. From (12) and (11), then follows (2). Q.E.D.

IV. Numerical Example

The application of (2) is quite convenient when a computational procedure for evaluation of the resonant frequency of the shielded resonator is available. The same computational procedure can be used afterwards to find Q_c due to resistive losses in the cavity walls. The method will be illustrated on the example of the $TE_{01\delta}$ dielectric resonator mounted on a dielectric substrate above the metal plane, such as shown in Fig. 2. The resonant frequency of the system can be found by a simple approximate procedure [8] which uses only the elementary functions available on pocket calculators. Suppose we want to evaluate the degradation of the unloaded Q_0-factor, due to the presence of losses in the ground conductor.

In the procedure [8], the resonant frequency is not available explicitly. The dimensions L_2 and D, the material properties ϵ_r and ϵ_2, and the frequency f_0 must be given at the outset of the computation, and the procedure gives L as a result. The variables of importance are thus related as follows:

$$L = L(L_2, f_0). \tag{13}$$

The change $\Delta f_0(\delta)$ required in (2) cannot be computed explicitly, so the solution must be found indirectly. For small changes of L_2 and f_0, the differential of (13) is

$$\Delta L = \frac{\partial L}{\partial L_2} \Delta L_2 + \frac{\partial L}{\partial f_0} \Delta f_0. \tag{14}$$

Since the resonator length is constant, $\Delta L = 0$, and the following expression can be used to find the desired derivative of frequency with respect to L_2:

$$\frac{\Delta f_0}{\Delta L_2} = -\frac{\partial L / \partial L_2}{\partial L / \partial f_0}. \tag{15}$$

The derivatives on the right-hand side may be evaluated numerically from (13), by making small increases (e.g., 0.1 percent) in L_2 and in f_0 and finding the corresponding changes in L. Using

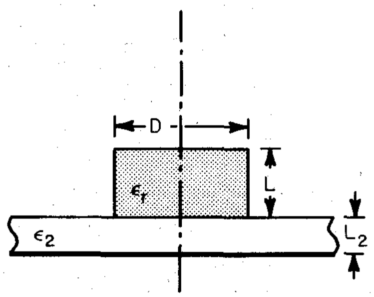

Fig. 2. Dielectric resonator mounted on a substrate.

(15), Q_c is then found from

$$Q_c = \frac{f_0}{\frac{\Delta f_0}{\Delta L_2} \cdot \delta}. \tag{16}$$

In the example to be computed, the resonator is specified by $\epsilon_r = 38$, $D = 10.5$ mm, $L = 4.6$ mm, and the dielectric substrate is specified by $\epsilon_2 = 2.5$ and $L_2 = 0.762$ mm. Suppose that the unloaded Q, given by the manufacturer, is $Q_0 = 5000$. This Q-factor takes into account the dielectric losses of the resonator material only.

As a first step in the computation, the resonant frequency is evaluated from (13) by iteration, the result being $f_0 = 5.483$ GHz. Actually, the accuracy of method [8] is only about 2 percent, but we have to keep a sufficient number of digits in order to evaluate the derivatives from finite differences. If frequency is now increased by 0.1 percent, the increment in L is found to be $\Delta L = 0.01254$ mm. If L_2 is next increased for 0.1 percent, the corresponding increment is $\Delta L = .5925$ μm. Thus, the required derivative computed by (15) is

$$\frac{\Delta f_0}{\Delta L_2} = -0.3429 \text{ GHz/mm}. \tag{17}$$

The sign is negative, because frequency decreases when L_2 is increased. To apply (2), the length L_2 has to be shortened by δ, i.e., increased by $-\delta$ so that the computed Q-factor comes out to be positive. From (1), the skin depth for a copper conductor is found to be $\delta = 0.8913$ μm, which then gives $Q_c = 17938$. Therefore, due to the contribution of conductor losses in the ground plane, the overall unloaded Q of the resonator in Fig. 2 will drop from 5000 to $(17938^{-1} + 5000^{-1})^{-1} = 3910$.

V. Corollaries

i) The derivation of (2) is valid only if the stored electric energy is stationary when the walls are receded for distance δ. Therefore, the field distribution must be of such a nature that the electric field normal to the shielding walls is zero. None of the HEM modes possess this property.

ii) The metal enclosure must possess the rotational symmetry. A box shaped as a parallelepiped would create normal components of the electric field on the walls, thus violating the assumption $\Delta W_e = 0$.

iii) Equation (2) is not an approximation, but it represents an exact relationship, as long as the assumptions utilized in the proof remain valid.

iv) The presence of the dielectric core inside the resonator is not required for the application of the incremental frequency rule. This may be seen by deriving an analytical expression for the Q-factor of a TE_{omp} hollow cylindrical resonator by evaluating the differential Δf in (2), instead of performing the conventional integration of the dissipated power along the walls. The result is identical with the published Q-factor value for hollow cylindrical resonator [9].

v) Since the quantities appearing in (2) can be observed by an experiment, the incremental frequency rule may be used in measurements.

vi) A partial Q-factor due to only one of the metal walls may be studied by receding only that wall and leaving the other walls intact. As an example, it is possible to analyze the effect of the metal tuning plunger on the Q-factor of the cavity, if the plunger is inserted into a cavity in such a way that the rotational symmetry is preserved.

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Loss Measurements of Nonradiative Dielectric Waveguide

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***Abstract*—A technique has been developed for precisely measuring the attenuation constant of the nonradiative dielectric waveguide (NRD-guide) at 50 GHz. The novelty of the present technique lies in incorporating the NRD-guide directional coupler into the measurement system and taking advantage of the total reflection of waves at the truncated end of the dielectric strip to facilitate the construction of the setup and to attain a high degree of accuracy in measurements. Measured attenuation constants were found to be about 13 dB/m for a polystyrene NRD-guide and 4 dB/m for a Teflon NRD-guide. These values indicate that the NRD-guide can be of practical use as a waveguide for millimeter-wave integrated circuits because of its low-loss nature as well as its radiation suppression capability. Calculation is also carried out in order to support measurements.**

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